

# Review on Unit Commitment under Uncertainty Approaches

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**Abstract – Wind power has already become an important renewable energy resource in many regions of the world. Because of its variability and uncertainty, integration of wind power presents a challenge that, if not adequately addressed, can jeopardize the operational reliability of a power system. Generally, generation unit commitment decisions are made once a day, i.e., the commitment decisions are made 24 or more hours ahead of the actual operation. Taking into account the uncertainty of wind power prediction, these decisions need to provide sufficient flexibility at a minimum price. This paper describes the current practice and analyzes unit commitment formulations available in literature highlighting their advantages and shortcomings.**

## I. INTRODUCTION

The primary concern in operating an electrical power system is to meet the demand for electricity at all times and under different conditions depending on the season, the climate, and the weather.

Modern power systems are supposed to accommodate large total capacity of distributed, volatile generation, as well as large-scale price responsive demand and electric vehicles which dramatically increases both supply and demand uncertainty [1]-[3]. Because of its variability and uncertainty, wind generation impacts power system operation and can potentially jeopardize its reliability. To deal with the larger uncertainty on the net load (the difference between electricity demand and the output of non-dispatchable generation), power system operators are increasing the reserve margins, thus increasing the regulation cost [4].

In order to minimize the operating cost of non-dispatchable resources, it is essential to derive a computationally effective approach to optimally select the units and their output level to preserve the operational reliability of the system. Unit commitment (UC), one of the most critical decision processes, is an optimization problem that generates the outputs of all the generators in a way that minimizes the system-wide fuel cost. Features included in most modern unit commitment models include generator minimum and maximum generation limits, ramping limits, minimum up and down time constraints, time-dependant start up costs and transmission capacity limits [5]-[8].

During the normal operation, system operator dispatches the committed generation resources to satisfy the actual demand and reliability requirements. In the event of a significant deviation between the actual and the expected system condition, system operator needs to take corrective actions, such as committing expensive fast-start generators, voltage regulation or load shedding, to maintain system security. The main causes of the unexpected events come from the uncertainties associated with the load forecast error, changes of system interchange schedule, and unexpected transmission and generation outages. [9]

Deterministic UC formulation is a traditional solution in which the net load is modeled using a single forecast for each wind farm output and the associated uncertainty is handled using ad-hoc rules, i.e., the generating units are committed to meet the deterministic forecast and the uncertainty is handled by imposing reserve requirements [10]-[13]. Such an approach is easy to implement in practice, but the ad-hoc rules do not necessarily adequately account for this uncertainty. Namely, committing extra generation resources for reserve is economically inefficient, while the power system may still suffer from capacity inadequacy in case of a significant deviation between real-time and expected net load. There is a lot of research on optimizing the reserve requirements based on deterministic criteria [14]-[17]. In [14] a new technique to determine the SR requirements at each period of the optimization horizon is proposed using a cost/benefit analysis. Similarly, in [15] the cost of interruptions is considered when optimizing the scheduling of spinning reserve. In [16] a probabilistic analysis of the reserve requirements is taken into account. The authors of [17] show that reserve requirements cannot be specified a priori without sacrificing the optimality.

A more rigorous approach is incorporating uncertainty in the unit commitment model itself, which is the focus of this review paper. Section II describes stochastic unit UC, Section III robust UC formulation, while Section IV describes interval UC formulation. Section V describes some recent advancement in hybrid UC models that combine the aforementioned formulations. Conclusions are duly drawn in Section VI.

## II. STOCHASTIC UNIT COMMITMENT

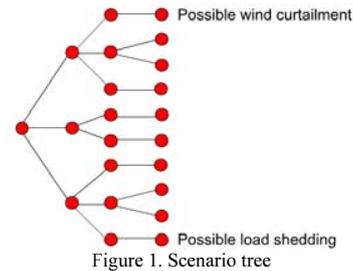
Stochastic UC is based on probabilistic scenarios. A finite set of scenarios is generated and assigned weight in proportion to their likelihood. Stochastic UC is

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formulated as a two-stage problem that determines the generation schedule that minimizes the expected cost over all of the scenarios respecting their probabilities. The commitment decisions are unique over all the scenarios, while dispatch decisions are scenario dependent. Including a large number of scenarios in the model requires computationally demanding simulations. Computational burden of the stochastic UC is dramatically increased with the time horizon as well, which is visualized in Figure 1. Thus, scenario reduction techniques that eliminate scenarios with very low probabilities and aggregate close scenarios are developed [18]. Similar scenarios get aggregated based on a particular metrics, such as their probability, hourly magnitude, or the resulting cost [19]. In [19] the authors did a comparison of scenario reduction techniques for the stochastic unit commitment. A clustering method, k-means is used to partition a given set of scenarios into a given number of clusters. The cluster features similar scenarios and is represented by a scenario with the lowest probability distance from the centroid. The centroid is an average pattern of all the scenarios from the cluster [19] [28]. The forward scenario selection and backward scenario reduction approaches are based on minimizing the Kantorovich distance between the scenarios in the original and in the reduced set [19] [29]. The forward scenario selection approach is used to construct a reduced set containing a desired number of scenarios by iteratively adding a scenario from the original set. Similarly, the backward reduction approach gives a reduced set by iteratively eliminating one scenario from the original set until the desired number of scenarios remains. Importance-sampling scenario reduction technique is used to select the scenarios that best represent the monetary impact of uncertainty on the operating cost [19] [30]. However, insufficient number of scenarios reduces accuracy of the solution and increases its cost. The eliminated scenarios may have great impact on the system, so stochastic UC formulations provide only probabilistic guarantees to the system reliability. It is important to note that the stochastic UC solution contains a certain amount of unhedged uncertainty, i.e. load shedding or wind curtailment in the most extreme scenarios might be cheaper than modifying the schedule to serve the net load over all the scenarios. Due to increased uncertainty in later hours of the time horizon, the amount of unhedged uncertainty increases over time [20]. In order to secure the robustness of the solution, a large set of scenarios is required, which is computationally demanding. Problems to be considered:

- Possible loss of information
- Disregarding the scenarios with comparatively low probability but great impact on the power system
- Availability of data
- Difficulties to identify accurate probability distribution of the uncertainty



In [21] the authors consider a set of possible scenarios rather than solving the UC problem for one expected and the worst-case demand scenario. Each of the scenarios is assigned a weight that reflects the possibility of its occurrence in the future. The solution must satisfy the constraint that if two different scenarios  $s$  and  $s'$  are indistinguishable at time period  $t$  based on the available information at time period  $t$ , the decision made for scenario  $s$  must be the same as that for scenario  $s'$ . The constraint is modeled by partitioning the scenario set at each time period into disjoint subsets called scenario bundles. Mathematically, a bundle at time period  $t$  is represented as a constraint on the decision variables of its scenarios. The objective function is to minimize weighted sum of the objective functions of the smaller problems, i.e. to minimize the expected cost over all of the scenarios. The problem can then be solved using a Lagrangian relaxation type of technique.

In [18] authors present a security-constrained unit commitment algorithm considering the volatility of wind power generation. To capture volatility it is assumed that the wind power is subject to a normal distribution  $N(\mu, \sigma^2)$  with forecasted wind power as expected value  $\mu$  and a percentage of  $\mu$  as its volatility ( $\sigma$ ). The Monte Carlo simulation is used to generate a large number of scenarios subject to the normal distribution. The probability assigned to each scenario is one divided by the number of generated scenarios. To decrease the computational requirement for large number of scenarios, a scenario reduction technique is used. The algorithm is formulated as an optimization problem with the objective function composed of fuel costs and startup and shutdown costs of generating units over the scheduling horizon. The problem is a large-scale mixed-integer non-linear program. The Benders' decomposition is applied to decompose the problem into master problem, feasibility check subproblems, and network security check subproblems. The master problem provides a commitment and dispatch solution that minimizes the operating cost of dispatchable units. The feasibility check subproblems whether the commitment and dispatch solution of the master problem can accommodate the volatility of the wind power in individual scenarios. The paper shows that the physical limitations of units, such as ramping, are crucial for accommodating the volatility of the wind power generation.

In [22] the authors present a stochastic model for the long-term solution of security-constrained UC. Forced outages of generating units and transmission lines are modeled as independent Markov processes, and load

forecasting uncertainties as uniform random variables. Optimization problem is decomposed into deterministic long-term subproblems. A scenario reduction method is used to obtain a tractable solution. A 6-bus system, the IEEE 118-bus system, and an 1168-bus system are used to test the algorithm. The stochastic solution provides more reliable decisions on energy allocation, fuel consumption, emission allowance, and long-term utilization of generating units in comparison to the deterministic UC solution.

### III. ROBUST UNIT COMMITMENT

Robust unit commitment formulations require a deterministic set of uncertainty, rather than a probability distribution on the uncertain data. Robust UC model described in [9] is a two-stage model: the first stage finds the optimal commitment decision, while the second stage generates the worst case dispatch cost under a fixed commitment solution from the first stage. The range of uncertainty is defined by the upper and lower bounds on the net load at each time period.

The robust model generates the optimal solution feasible for all realizations of the uncertain data within the given bounds. By minimizing the highest cost over all realizations, the model tends to provide conservative solutions, thus more expensive, which can be adjusted using the budget of uncertainty. The budget of uncertainty is defined as the number of buses that are allowed to deviate from a given central wind forecast in the worst case scenario [9]. The higher the budget of uncertainty the more robust the solution. The value for the budget of uncertainty is not known in advance but depends on the system and the experience. ISO New England (NE) enlisted Lawrence Livermore National Laboratory (LLNL)'s help in determining whether a robust UC would improve system reliability while keeping the operation cost relatively low in the presence of renewable variability [31]. In the study, a comprehensive evaluation of robust UC was conducted. The objectives were to identify the optimal conservatism level to balance the economic efficiency and operational reliability of robust UC solutions, as well as to compare the robust and deterministic approaches.

In [23] authors present a two-stage network constrained robust UC problem introducing a two-dimensional uncertainty set to describe the uncertain problem parameter, allowing the uncertainty correlations among different buses and among different time periods. A bilinear separation approach generates tight lower and upper bounds for the optimal objective value and it is tested for computational efficiency on a 118-bus system. The authors use the Benders' decomposition that includes feasibility and optimality cuts. A case in which the demand at each bus in each operating hour may be uncertain is addressed, and the uncertainties are described by a given polyhedral uncertainty set rather than by the probability distribution.

In [24] a two-stage robust UC model is developed to obtain day-ahead generator schedules where wind uncertainty is captured by a polytopic uncertainty set. The uncertainty set modeling method captures the random

nature of wind without any explicit description of the distribution function. The model is also extended to include the demand response strategy. The authors performed a computational study on an IEEE 18-bus system to show that the robust UC model can utilize wind generation and lower overall generation cost.

A robust UC model that takes into account the worst-case scenario of wind power output with deterministic loads during all periods is presented in [25]. This approach distributes the random problem parameters in a predetermined uncertainty set containing the worst-case scenario. Uncertain wind power output in each time period is within an interval defined by its lower and upper bounds which are obtained based on historical data or estimated with a confidence interval. The problem is formulated as a two-stage min-max problem with the objective to minimize the total cost under the worst wind power output scenario. The degree of conservatism is adjusted using the budget of uncertainty, an integer that takes a value between 0 and the number of hours in the time horizon  $T$ , to restrict the number of time periods that allow the actual wind power output to deviate from its forecasted value. By adjusting the value of the budget of uncertainty, system operators can control the robustness of the solution. The higher the budget of uncertainty, the more robust the solution. The UC decisions are made at the first stage, while the second stage results in economic dispatch. Wind power generation values are subject to uncertainty, and they are presented by random variables described by the uncertainty set. The authors describe their solution methodology and test the algorithm on a 6-bus and a modified 118-bus system. The wind power uncertainty is additionally hedged using pumped storage hydro units.

### IV. INTERVAL UNIT COMMITMENT

Interval UC formulations produce a schedule that minimizes the cost of serving the most probable net load forecast while guaranteeing feasibility in the entire uncertainty range that is delimited with upper and lower bounds as in robust unit commitment formulations. Figure 2. shows the central forecast, i.e. the most probable realization to be minimized in the objective function, along with the upper and lower bounds and the transitions between them. The solution is optimal along central forecast while remaining feasible along upper and lower bound. The solutions tend to be conservative because of the steep ramp requirements that need to be satisfied in between all consecutive time periods, as shown in Figure 2. The formulation is computationally more efficient than the stochastic unit commitment formulation because the model can be formulated using three scenarios, i.e. the central forecast and the upper and lower bounds, while the transition constraints are modeled as constraints. The interval UC can also be formulated as a two-stage problem where the optimal solution is found in the first stage and then tested for feasibility in the second stage.

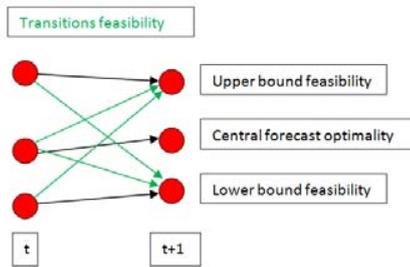


Figure 2. Interval unit commitment

A method for daily UC and dispatch incorporating wind power based on the interval number theory is introduced in [26]. Uncertain wind power generation is represented by a functional interval. The optimal model is first divided into two deterministic mix-integer programming subproblems with the parameters expressed as constants. The interval solutions of the model can be constructed using the solutions of two subproblems. The model is tested on a 30-bus system showing that the proposed method can be used for the unit commitment. The forecasting accuracy has a great impact on the optimal interval of UC.

## V. HYBRID UNIT COMMITMENT MODELS

Recently, some authors have developed hybrid models that exploit the advantages and eliminate disadvantages of the models presented in the previous Sections. Such models are unified stochastic and robust unit commitment formulation proposed in [27] and hybrid stochastic/interval unit commitment model proposed in [20].

### A. Unified stochastic and robust unit commitment

Stochastic UC formulations face computational challenges due to the large scenario size necessary to secure system robustness, while the robust UC formulations tend to result in conservative, thus expensive solutions. To take advantage of both of the approaches the authors of [27] propose a unified stochastic and robust UC model able to achieve low expected total cost while ensuring the system robustness. The objective function contains stochastic and robust parts that are weighed with scaling factors which can be adjusted by system power operators. The model generates a less conservative solution as compared to the two-stage robust optimization approach and a more robust solution as compared to the two-stage stochastic optimization approach. As in previous two-stage models, at the first stage the day-ahead unit commitment decisions are made. The second stage decides on the dispatch for each scenario for the stochastic optimization part and the worst-case scenario for the robust optimization part. A new parameter is introduced ranging between 0 and 1 to represent the weight of the worst case generation cost. The authors tested their approach on a modified IEEE 118-bus system.

The downside is that this approach employs heuristics to balance the stochastic and robust unit commitment solutions, which may result in suboptimal solution.

### B. Hybrid stochastic/interval unit commitment

The authors of [20] propose a model that applies the stochastic formulation to the initial operating hours of the optimization horizon and then switches to the interval formulation for the remaining hours. The switching time is optimized to achieve optimal trade-off between the cost of unhedged uncertainty from the stochastic UC and the security premium of the interval UC. The two formulations are applied sequentially, instead of simultaneously according to their heuristically chosen weights in [27]. The stochastic UC, which is more cost effective but for a computationally tractable numbers of scenario less robust, is applied to the first part of the time horizon during which the wind power output predictions are more accurate. The model then switches to interval UC offering more robust solution for the remaining time period when the uncertainty is also greater.

The authors introduce a day-ahead cost (DAC) which represents the expected operating cost at the day-ahead stage. The DAC of a stochastic UC formulation is a minimum expected operating cost over the set of scenarios, while DAC of the interval UC formulation represents the cost of the schedule that minimizes the cost of meeting the central net load forecast while ensuring the feasibility of the predefined worst-case scenarios. A more conservative uncertainty model results in greater DAC because of a more conservative schedule. The actual operating cost (AOC) is the cost that includes the corrective actions which include redispatch of committed generators and starting up or shutting down other generators to account for deviations between actual and predicted output. This hybrid UC formulation aims to minimize AOC by finding the optimal balance between the day-ahead security cost and the expected cost of uncertainty associated with the day-ahead schedule. The authors present a method to optimally select the switching time and they test their approach on a modified version of the 24-bus system

## VI. CONCLUSION

The common goal of UC formulations is to minimize the operating cost, while ensuring sufficient reserve to accommodate real-time realization of uncertainty. The main difference between models is the representation of this uncertainty. The stochastic UC formulation tends to give the most cost effective solution. However, in order to secure the robustness of solution, a large number of scenarios is required which renders this formulation computationally intensive. The robust and interval UC formulations secure a robust solution, but tend to give conservative, thus more expensive solutions. The hybrid models are developed to exploit the best traits of stochastic formulation (being the most cost effective) and robust or interval formulation (securing the robustness).

A research direction in unit commitment under uncertainty will include a combination of the existing methods that will try to exploit their respective

advantages. However, an important issue is the scenario generation and reduction techniques, which is a field of the ongoing research.

Also, in the scientific community it is still not clear how to validate the solutions obtained using different techniques. Although Monte Carlo simulations are the most common technique for classification of the solutions, it is sensitive to its input parameters (historical errors). This method should also be re-examined and improved.

#### REFERENCES

- [1] A. Ipakchi and F. Albuyeh, "Grid of the future," *IEEE Power Energy Mag.*, vol. 7, no. 2, pp. 52-62, Mar/Apr. 2009.
- [2] J. G. Vlachogiannis, "Probabilistic constrained load flow considering integration of wind power generation and electric vehicles," *IEEE Trans. Power Syst.*, vol. 24, no. 4, pp. 1808-1817, Nov. 2009.
- [3] Y. Wang, Q. Xia, and C. Kang, "Unit commitment with volatile node injections by using interval optimization," *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1705-1713, Aug. 2011
- [4] J. Kiviluoma et al., "Impact of wind power on the unit commitment, operating reserves, and market design," in *Proc. of IEEE Power and Energy Society General Meeting 2011*, San Diego, California, July 2011, pp. 1-8
- [5] H. Pandžić, T. Qiu, and D. Kirschen, "Comparison of state-of-the-art transmission constrained unit commitment formulations," in *Proc. of IEEE PES General Meeting 2013*, Vancouver, Canada, July 2013, pp. 1-5.
- [6] J. M. Arroyo and J. Conejo, "Optimal response of a thermal unit to an electricity spot market," *IEEE Tran. Power Syst.*, vol. 15, no. 3, pp. 1098-1104, Aug. 2000.
- [7] D. Rajan and S. Takriti, "Minimum up/down polytopes of the unit commitment problem with start-up costs," Jun. 2005, *IBM Research Report*
- [8] C. K. Simoglou, P. N. Biskas, and A. G. Bakirtzis, "Optimal self-scheduling of a thermal producer in short-term electricity markets by MILP," *IEEE Tran. Power Syst.*, vol. 25, no. 4, pp. 1965-1977, Nov. 2010.
- [9] D. Bertsimas, E. Litvinov, X. A. Sun, Z. Jinye, and T. Tongxin, "Adaptive robust optimization for the security constrained unit commitment problem," *IEEE Trans. Power Syst.*, vol. 28, no. 1, pp. 52-63, Feb. 2013
- [10] Y. V. Makarov, C. Loutan, M. Jian, and P. de Mello, "Operational impacts of wind generation on California Power Systems," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 1039-1050, May 2009.
- [11] Y. G. Rebours, D. S. Kirschen, M. Trotignon, and S. Rossignol, "A survey of frequency and voltage control ancillary services. Part I: Technical features," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 350-357, Feb. 2007.
- [12] Y. G. Rebours, D. S. Kirschen, M. Trotignon, and S. Rossignol, "A survey of frequency and voltage control ancillary services. Part II: Economic features," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 358-366, Feb. 2007.
- [13] ERCOT *Methodologies for determining ancillary service requirements*, 2012. [Online] Available at: [www.ercot.com/meetings/wms/keydocs/2004/0819/WMS08192004-3.doc](http://www.ercot.com/meetings/wms/keydocs/2004/0819/WMS08192004-3.doc)
- [14] M. A. Ortega-Vazquez and D. S. Kirschen, "Optimizing the spinning reserve requirements using a cost/benefit analysis," *IEEE Trans. Power Syst.*, vol. 22, no. 1, pp. 24-33, Feb. 2007.
- [15] M. A. Ortega-Vazquez, D. S. Kirschen, and D. Pudjanto, "Optimising the scheduling of spinning reserve considering the cost of interruptions," *IEEE Proc. Gener., Trans. Distr.*, vol. 153, no. 5, pp. 570-575, Sept. 2006.
- [16] F. D. Galiana, F. Bouffard, J. M. Arroyo, and J. F. Restrepo, "Scheduling and pricing of coupled energy and primary, secondary, and tertiary Reserves," *Proc. IEEE*, vol. 93, no. 11, pp. 1970-1983, Nov. 2005.
- [17] J. M. Arroyo and F. D. Galiana, "Energy and reserve pricing in security and network-constrained electricity markets," *IEEE Trans. Power Syst.*, vol. 20, no. 2, pp. 634-643, May 2005.
- [18] J. Wang, M. Shahidehpour, and Z. Li, "Security-constrained unit commitment with volatile wind power generation," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 1319-1327, Aug. 2008.
- [19] Y. Dvorkin, Y. Wang, H. Pandžić, D. Kirschen, "Comparison of scenario reduction techniques for the stochastic unit commitment", in *Proc. of IEEE PES General Meeting, Conference & Exposition 2014.*, National Harbor, Maryland, July 2014, pp. 1-5
- [20] Y. Dvorkin, H. Pandžić, M. Ortega-Vazquez, and D. S. Kirschen, "A hybrid stochastic/interval approach to transmission-constrained unit commitment," *IEEE Trans. Power Syst.*, early access.
- [21] S. Takriti, J. R. Birge, and E. Long, "A stochastic model for the unit commitment problem," *IEEE Trans. Power Syst.*, vol. 11, no. 3, pp. 1497-1508, Aug. 1996.
- [22] L. Wu, M. Shahidehpour, and T. Li, "Stochastic security-constrained unit commitment," *IEEE Trans. Power Syst.*, vol. 22, no. 2, pp. 800-811, May 2007.
- [23] R. Jiang, M. Zhang, G. Li, and Y. Guan, "Two-stage network constrained robust unit commitment problem," *Eur. J. Oper. Res.*, vol. 234, no.3, pp. 751-762, May 2014.
- [24] B. Zheng and L. Zhao, "Robust unit commitment problem with demand response and wind energy," in *Proc. IEEE PES General Meeting 2012*, San Diego, USA, July 2012, pp. 1-8.
- [25] R. Jiang, J. Wang, and Y. Guan, "Robust unit commitment with wind power and pumped storage hydro," *IEEE Trans. Power Syst.*, vol. 27, no. 2, pp. 800-810, May 2012.
- [26] X. Sun and C. Fang, "Interval mixed-integer programming for daily unit commitment and dispatch incorporating wind power," in *Proc. Power System Technology (POWERCON) 2010*, Hangzhou, China, Oct. 2010, pp. 1-6
- [27] C. Zhao and Y. Guan, "Unified stochastic and robust unit commitment," *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 3353-3376, Aug. 2013.
- [28] D. Arthur and S. Vassilvitskii, "K-means++: the advantages of careful seeding," in *Proc. of 18<sup>th</sup> Annual ACM-SIAM Symposium on Discrete Algorithms*, pp. 1027-1035, New Orleans, Louisiana, 2007.
- [29] J. Dupacova, N. Grove-Kuska, and W. Romisch, "Scenario reduction in stochastic programming: An approach using probability metrics," *Math. Program.*, Vol. 95, pp. 493-511, 2003.
- [30] A. Papavasiliou and S. S. Oren, "Multiarea stochastic unit commitment for high wind penetration in a transmission constrained network," *Operation Research*, Vol. 61, No. 3, pp. 578-592, 2013.
- [31] J. Zhao et al., "A comprehensive evaluation of robust unit commitment" 2014., [Online] Available at: <https://e-reports-ext.llnl.gov/pdf/770804.pdf>
- [32] H. Pandžić, A. Conejo, I. Kuzle, "An EPEC Approach to the Yearly Maintenance Scheduling of Generating Units," *IEEE Transactions on Power Systems*, Vol. 28, No. 2, pp. 922-930, 2013